

# DECISION-MAKING ON FUZZY PIECES OF EVIDENCE

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## Abstract

This paper identifies that a meaningful rule in a fuzzy expert systems (FES) depends to some degree on the number of rule antecedents connected to a rule consequent. The paper presents an alternative for scenarios where a larger number of rule antecedents applies to the same rule consequent. Results of an application of the method in the domain of coronary heart disease risk assessment (CHDRA) indicate the value of the method.

## 1. Introduction

FES rules are usually formulated as IF-THEN statements, with one or more antecedents connected to a consequent via operators like AND, OR, etc. (Figure 1).

(w) IF (Antecedent<sub>1</sub>) OP (Antecedent<sub>2</sub>) ... OP (Antecedent<sub>n</sub>) THEN (Consequent<sub>1</sub>)

Figure 1: General form of a FES rule,

where  $n \in \mathbf{N}$ , OP is standing for operators like AND, OR, etc., and w represents a weight value indicating the importance of a rule. Now, imagine a FES rule where 2 antecedents apply to the same consequent ( $n = 2$ ). Further, let Antecedent<sub>1</sub> be activated to a degree of 0.8, and Antecedent<sub>2</sub> to a degree of 0.7. The weight value shall be 1.0, and OP shall be an OR operator defined as: Consequent<sub>1</sub> = max[Antecedent<sub>1</sub>, Antecedent<sub>2</sub>]. In this situation Consequent<sub>1</sub> would be activated to a degree of max[0.8, 0.7] = 0.8. There is nothing extraordinary here. The process described is standard process [Ross, 1995]. Now imagine a more complex scenario where 20 antecedents apply to a Consequent<sub>1</sub> ( $n = 20$ ). A possible scenario may look like: Consequent<sub>1</sub> = max[0.7<sub>1</sub>, 0.8<sub>2</sub>, 0.6<sub>3</sub>, ... 0.3<sub>20</sub>] = 0.8, for example. In this situation we probably are less confident in the applicability and usefulness of the rule. The large number of 20 antecedents applying to the same consequent intuitively raises concerns. The formulation of rules showing such complexity however might be common in some domains. To approach the problem this paper presents a method that aims to include each activated rule antecedent more actively in the reasoning process of the FES.

Section 2 therefore describes the method. Section 3 provides results of an application of the

method in the CHDRA domain. Conclusions and future work end the paper in Section 4.

## 2. Fuzzy Expert System Decision-making

To explain the method we describe a simple FES where two inputs (Input<sub>1</sub>, Input<sub>2</sub>) relate to an output (Output<sub>1</sub>).

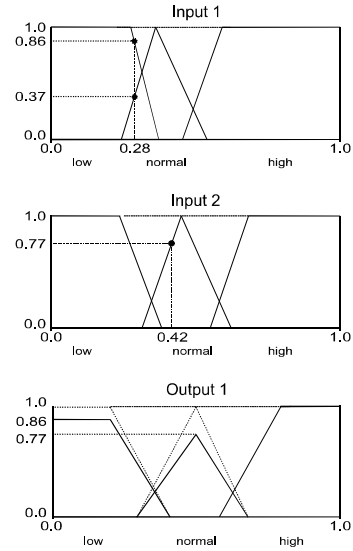


Figure 2: FES and fuzzy pieces of evidence.

The following two rules shall be included in the FES:

IF (Input<sub>1</sub>) IS (normal) OR (Input<sub>2</sub>) IS (normal)  
THEN (Output<sub>1</sub>) is (normal)  
IF (Input<sub>1</sub>) IS (low) OR (Input<sub>2</sub>) IS (low)  
THEN (Output<sub>1</sub>) is (low)

Figure 3: Rules included in the FES example.

Further, both rules shall carry the weight 1.0, and the OR operator employed shall be the same as it was used before. Figure 2 illustrates that the value 0.28 activates the fuzzy sets *low* and *normal* in Input<sub>1</sub> to a degree of 0.86 and 0.37, respectively, whereas the value 0.42 in Input<sub>2</sub> activates only the fuzzy set *normal* to a degree of 0.77. Figure 2 further shows: (a) how these activations apply to Output<sub>1</sub>, and (b) the defuzzification of Output<sub>1</sub>. There exist many possibilities for both processes [Cox, 1995]. Here, for example, an output fuzzy set is scaled according to the highest input activation, and

defuzzification selects the maximum activation in  $Output_1$  for subsequent decision-making. Based on these definitions the fuzzy set  $Output_{1/normal}$  is scaled down to  $\max[0.77, 0.37] = 0.77$ , and the fuzzy set  $Output_{1/low}$  to  $\max[0.86] = 0.86$ . Defuzzification of these values establishes the outcome:  $Output_1 = \max[Output_{1/low}/0.86, Output_{1/normal}/0.77] = Output_{1/low}/0.86$ . Note however that the sum of the two activations indicating a *normal* outcome ( $0.77 + 0.37 = 1.14$ ) is larger than the sum of the activation(s) indicating a *low* outcome (0.86)! In this situation it can be argued that the contribution  $Input_{1/normal}/0.37$  is not considered carefully by the chosen decision-making process. In the context of this research (**many** antecedents — same consequent) it seems to be even more an oversimplification to generate the outcome on a criteria that ‘simply’ selects a maximum activation from many other activations [Chen and Hwang, 1992]. Naturally the question arises: “Are there other ways to combine these pieces of evidence?”

## 2.1 Accumulating Fuzzy Pieces Of Evidence

The proposed method is based on 2 assumptions:

- (1) Every activation of an input fuzzy set is regarded to be a piece of (fuzzy) evidence supporting the domain knowledge an expert formulated via rules and fuzzy sets.
- (2) Each piece of evidence should be incorporated more actively in the decision-making process.

These assumptions are implemented in 3 steps:

**Step 1:** Evidence accumulation.

**Step 2:** Normalisation.

**Step 3:** Decision-making.

For example, an application of Step1, Step 2, and Step 3 on the example illustrated in Figure 2 leads to Table 1.

**Table 1:** Utilisation of Step 1, Step 2, and Step 3.

	<i>low</i>	<i>normal</i>	<i>high</i>
<b>Input<sub>1</sub></b>	0.86	0.37	—
<b>Input<sub>2</sub></b>	—	0.77	—
<b>Step 1: Accumulation</b>	0.86	1.44	—
<b>Step 2: Normalisation</b>	0.75	1.00	—
<b>Step 3: Decision-making</b>	$Output_1 = Output_{1/normal}/1.00$		

In Table 1 the accumulation of the pieces of evidence produces:  $Output_{1/low} = 0.86$ , and  $Output_{1/normal} = 0.37 + 0.77 = 1.44$ . Normalisation of these values generates:  $Output_{1/low} = 0.75$ , and  $Output_{1/normal} = 1.00$ . The method therefore produces the outcome:  $Output_1 = normal$ . Note that this result is different to the result in the previous section ( $Output_1 = Output_{1/low}/0.86$ )! Consequently, the questions arises: “Which method is the better decision-making method?” To answer this questions it has to be mentioned that the development of many FESs doesn’t follow strict rules. FES building is a ‘*trial and error*’

process where the system is ‘tuned’ by the developer towards the ‘best’ solution to the problem at hand. The right question therefore is: “Does the method work on a real problem?”

## 3. An Application Of The Method

In the domain of CHDRA cholesterol has been identified as one of the main risk factor for myocardial infarction and subsequent sudden death [Levy, 1993]. In a blood test a clinician first finds out what a subject’s TOTAL cholesterol level is. If this level is too high then further measurements of LDL and HDL cholesterol are required [Slyper, 1994]. The two ratios TOTAL/HDL and LDL/HDL are also important because they provide more meaningful indicators of coronary heart disease risk than TOTAL cholesterol per se. However, having the values for TOTAL, LDL, HDL, TOTAL/HDL, and LDL/HDL in front of him, for example, a clinician simply might say that a subject’s CHOLESTEROL in terms of CHD risk is *normal*. Further, for a clinician it may also be possible to assess individual cholesterol values. For example, a clinician may say that a LDL value of 1.50 mmol/l indicates *normal* CHOLESTEROL.

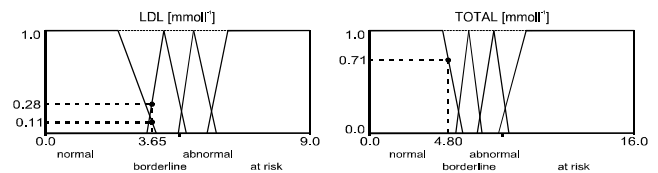
### 3.1 Medical Data And Domain Expert Tasks

The basis for an application is a data set consisting of 166 records. Collected in a wider CHDRA study a record includes the values for TOTAL, LDL, HDL, TOTAL/HDL, and LDL/HDL of a subject [Lopes *et al*, 1994]. For this study a domain expert was initially asked to assess the CHOLESTEROL of each subject, via indicating one of the fields (*normal*, *borderline*, *abnormal*, or *at risk*) for each record (Table 2).

**Table 2:** Domain expert’s assignment on 166 data records.

<b>CHOLESTEROL / Expert’s Decision</b>			
<i>normal</i> / 79	<i>Borderline</i> / 61	<i>abnormal</i> / 24	<i>at risk</i> / 2

For example, according to the expert there are 79 records indicating *normal* CHOLESTEROL. The expert then was asked to define fuzzy sets for the different cholesterol types and ratios. For simplicity Figure 4 only illustrates fuzzy sets for LDL, and TOTAL cholesterol.



**Figure 4:** Fuzzy sets for LDL, and TOTAL cholesterol.

Figure 4 also shows a possible activation by the values  $LDL = 3.65$  mmol/l, and  $TOTAL = 4.80$  mmol/l. The activations generated by these values can be used by the method for a simulation of the expert’s decision-making.

### 3.2 Results

First results are established by a comparison of the assignment given by the expert on the 166 records, and the assignment generated by the method on the same records (Table 3).

**Table 3:** Comparison between expert and method assignment.

Record	Proposed method*				Expert
	N	B	AB	AR	
1	0.48	<b>1.00</b>	0.27	0.00	B
...	...	...	...	...	...
166	<b>1.00</b>	0.80	0.40	0.00	N

Total number of matching outcomes = 137 = 82.5%

\* N = normal, B = borderline, AB = abnormal, AR = at risk.

For example, the expert's assignments on the first and the last record in Table 3 are *borderline* (B) and *normal* (N). For both records the method determines the same categories (record 1 = 1.00 = **B**, and record 166 = 1.00 = **N**). The total number of 137 (82.5%) matching outcomes presented in Table 3 can be taken as a first indicator for the value of the method. There is one more interesting point worth mentioning. For some records it was difficult for the expert to come up with a confident assignment, because of his opinion that a record belongs to the boundary region between two categories. In these situations the expert was more or less forced to choose one of the available categories. On the other hand, the proposed method is able to identify such records. For example, the last record in Table 3 shows similarly high values for *normal* (1.00), and *borderline*. (0.80) CHOLESTEROL. This may indicate that the subject's CHOLESTEROL is located in the boundary region between these two categories. The crisp decision N = *normal* given by the expert does not reflect this fact.

### 4. Conclusions And Future Work

The paper proposed a method for FESs where many rule antecedents apply to the same rule conclusion, and where the application of conventional processes (e.g. maximum OR) seems to be less meaningful. The advantages of the method are:

- The method is intuitively appealing, and easy to implement.
- The decision-making process is transparent, and explainable to a system user.
- The method allows the identification of values falling in-between two categories.
- The method can be easily extended for multiple expert decision-making scenarios.

The method was tested in the CHDRA/CHOLESTEROL domain. The relatively high number of correct results achieved in the study indicates the value of the method. Future work aims for a mathematical framework for the method. The framework intends to make the method applicable for decision-making scenarios including multiple knowledge sources (e.g., multiple experts).

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